التفاضل والتكامل ٢ ريض ١١١٣

كلية الحاسب جامعة الامام

مذكرة الواجبات

شرح الأستاذ / أبو عمر سلومة

جوال / ٠٥٩٠٦٤٧١٧١

شرح الواجبات بموقع الأستاذ أبو عمر

قواعد التكامل

For any rational power $r \neq -1$,

$$\int x^r dx = \frac{x^{r+1}}{r+1} + c.$$

In any interval not containing 0,

$$\int \frac{1}{x} dx = \ln |x| + c.$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c, \text{ in any interval in which } f(x) \neq 0.$$

Suppose that $f(x)$ and $g(x)$ have antiderivatives. Then, for any constants, a and b,

$$\int [af(x) + bg(x)]\, dx = a \int f(x)\, dx + b \int g(x)\, dx.$$

$$\int x^r dx = \frac{x^{r+1}}{r+1} + c, \text{ for } r \neq -1 \text{ (power rule)}$$

$$\int \sin x\, dx = -\cos x + c$$

$$\int \cos x\, dx = \sin x + c$$

$$\int \sec^2 x\, dx = \tan x + c$$

$$\int \csc^2 x\, dx = -\cot x + c$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$$

$$\int \sec x \tan x\, dx = \sec x + c$$

$$\int \csc x \cot x\, dx = -\csc x + c$$

$$\int e^x dx = e^x + c$$

$$\int e^{-x} dx = -e^{-x} + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$$

$$\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + c$$

Integration

EXERCISES 4.1

In exercises 5–28, find the general antiderivative.

القاعدة $\rightarrow \int a x^{n} dx = \frac{a x^{n+1}}{n+1} + c$

8. $\int \left(2x^{-2} + \frac{1}{\sqrt{x}}\right) dx$

$= \int (2x^{-2} + x^{-\frac{1}{2}}) dx$

$= \frac{2}{-1} x^{-1} + 2 x^{\frac{1}{2}} + c = -2x^{-1} + 2x^{\frac{1}{2}} + c$

10. $\int \frac{x + 2x^{3/4}}{x^{5/4}} dx$

$= \int \left(\frac{x}{x^{\frac{5}{4}}} + \frac{2x^{\frac{3}{4}}}{x^{\frac{5}{4}}} \right) dx = \int (x^{1-\frac{5}{4}} + 2x^{\frac{3}{4}-\frac{5}{4}}) dx$

$= \int (x^{-\frac{1}{4}} - 2x^{-\frac{1}{2}}) = \frac{4}{3} x^{\frac{3}{4}} + 4x^{\frac{1}{2}} + c$

13. $\int 2 \sec x \tan x \, dx$

$= 2 \sec x + c$

لسنا الافضل لكن نسعى نحو A+

القاعدة $\rightarrow \int \sec x \tan x \, dx = \sec x + c$

14. $\int \frac{4}{\sqrt{1-x^2}}\,dx = 4\sin^{-1}x + C$

القاعدة: $\rightarrow \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right)+c$

لاحظ

15. $\int 5\sec^2 x\,dx$ القاعدة: $\rightarrow \int \sec^2 x\,dx = \tan x + c$

$= 5\tan x + C$

لاحظ

16. $\int 4\frac{\cos x}{\sin^2 x}\,dx$

لاحظ

$= \int 4 \frac{\cos x}{\sin x \sin x}\,dx$

$= \int 4 \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}\,dx = \int 4 \cot x \csc x\,dx$

$= -4\csc x + C$

القاعدة: $\rightarrow \int \csc x \cot x\,dx = -\cot x + c$ لاحظ

In exercises 5–28, find the general antiderivative.

19. $\int (3\cos x - 1/x)\,dx$

القاعدة $\int \cos(ax)\,dx = \frac{1}{a}\sin ax + c$

$= 3\int \cos x\,dx - \int \frac{1}{x}\,dx = 3\sin x - \ln|x| + c$

21. $\int \frac{4x}{x^2+4}\,dx$

القاعدة $\rightarrow \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

$= 2\int \frac{2x}{x^2+4} = 2\ln|x^2+4| + c$

22. $\int \frac{3}{4x^2+4}\,dx$

القاعدة $\rightarrow \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{3}{4}\int \frac{dx}{x^2+1}\,dx = \frac{3}{4}\tan^{-1}x + c$

24. $\int \left(2\cos x - \sqrt{e^{2x}}\right)dx$

القاعدة $\rightarrow \int e^{ax}\,dx = \frac{e^{ax}}{a} + c$

$= 3\int \cos x\,dx - \int \sqrt{(e^x)^2}\,dx$

$= 3\int \cos x\,dx - \int e^x\,dx = 3\sin x - e^x + c$

ضرب

28. $\int x^{2/3}(x^{-4/3} - 3)\, dx$

$= \int x^{\frac{2}{3}} x^{\frac{-4}{3}} dx - 3\int \cancel{x}^{\frac{2}{3}} dx$

$= \int x^{\frac{-2}{3}} dx - 3\int x^{\frac{2}{3}} dx = \frac{3}{1}x^{\frac{1}{3}} - 3\cdot\frac{3}{5}x^{\frac{5}{3}} + C$

In exercises 29 and 30, find the derivative.

30. $\dfrac{d}{dx} \ln|\sin x - 2|$

$\frac{d}{dx} \ln|\sin x - 2| = \frac{1}{\sin x - 2} \frac{d}{dx}(\sin x - 2)$

$= \dfrac{\cos x}{\sin x - 2}$

EXERCISES 4.5

In exercises 1–18, use Part I of the Fundamental Theorem to compute each integral exactly.

$\int_a^b f(x)\,dx = F(b) - F(a)$

4. $\int_0^2 (x^3 + 3x - 1)\,dx$

$\int a x^n\,dx = \frac{a x^{n+1}}{n+1} + c$

$= \left(\frac{x^4}{4} - 3\frac{x^2}{2} - x\right)_0^2$

$= \left(\frac{2^4}{4} - 3\frac{2^2}{2} - 2\right) - 0 = 4 - 6 - 2 = -4$

6. $\int_1^2 \left(4x - \frac{2}{x^2}\right) dx$

$\int a x^n\,dx = \frac{a}{n+1} x^{n+1} + c$

$= \int_1^2 (4x - 2x^{-2})\,dx = \left(\frac{4x^2}{2} - \frac{2x^{-1}}{-1}\right)_1^2$

$= \left(2x^2 + \frac{2}{x}\right)_1^2 = \left(2(4) + \frac{2}{2}\right) - (2+2) = 9 - 4 = 5$

8. $\int_0^2 \left(\frac{e^{2x} - 2e^{3x}}{e^{3x}}\right) dx$

$\int e^{ax}\,dx = \frac{e^{x}}{a} + c$

$= \int_0^2 \left(\frac{e^{2x}}{e^{3x}} - 2\frac{e^{3x}}{e^{3x}}\right) dx$

$= \int_0^2 (e^{-x} - 2)\,dx = (-e^{-x} - 2x)_0^2$

$= (-e^{-2} - 4) - (-e^{0}) = -e^{-2} - 4 + 1 = -\frac{1}{e^2} - 3$

10. $\int_{\pi/4}^{\pi/2} 3\csc x \cot x \, dx$

$\int \csc x \cot x \, dx = -\csc x + c$

$\sin\frac{\pi}{2} = 1 \ , \ \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$= 3\left(-\csc x\right)_{\frac{\pi}{4}}^{\frac{\pi}{2}} = -3\csc\frac{\pi}{2} + 3\csc\frac{\pi}{4}$

$= -3 + 3\sqrt{2}$

13. $\int_0^{1/2} \frac{3}{\sqrt{1-x^2}} \, dx$

$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

$= 3\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx = 3\sin^{-1} x \Big|_0^{\frac{1}{2}} = 3\left(\sin^{-1}\frac{1}{2} - \sin^{-1} 0\right)$

$= 3\left(\frac{\pi}{6} - 0\right) = \frac{\pi}{2}$

14. $\int_{-1}^{1} \frac{4}{1+x^2} \, dx$

$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$4\int_{-1}^{1} \frac{1}{1+x^2} dx = 4\left(\tan^{-1} x\right)_{-1}^{1} = 4\left(\tan^{-1} 1 - \tan^{-1} -1\right)$

$= 4\left(\frac{\pi}{4} + \frac{\pi}{4}\right) = 4\left(\frac{\pi}{2}\right) = 2\pi$

18. $\int_0^t (\sin^2 x + \cos^2 x) \, dx$

$= \int_0^t 1 \, dx = (x)_0^t$

$= t$

$\sin^2 x + \cos^2 x = 1$

In exercises 19–24, find the given area.

20. The area below the x-axis and above $y = x^2 - 4x$

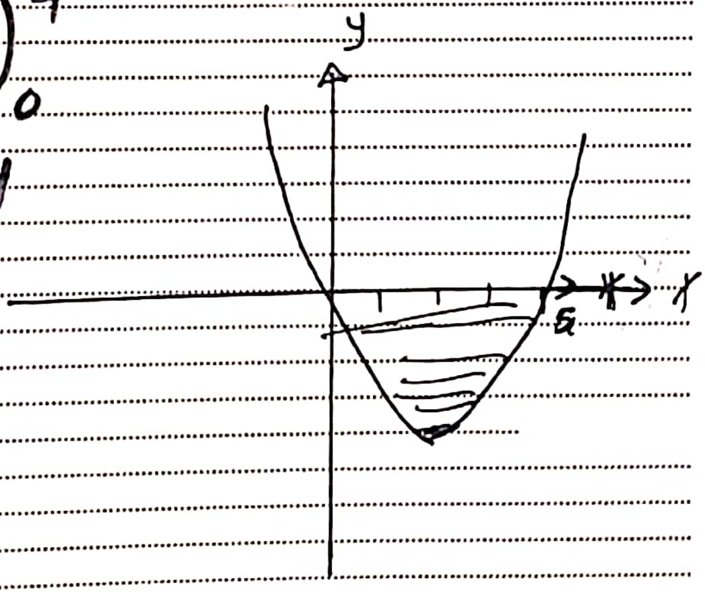
$x^2 - 4x = 0 \quad x(x-4) = 0$

$x = 0 \quad , \quad x = 4$

$\text{Area} = \int_0^4 -(x^2 - 4x)\,dx = \int_0^4 (4x - x^2)\,dx$

$= \left(\frac{4x^2}{2} - \frac{x^3}{3}\right)\Big|_0^4$

$= 2x^2 - \frac{x^3}{3}\Big|_0^4$



$= 2(4)^2 - \frac{4^3}{3}$

$= 32 - \frac{64}{3}$

$= \frac{96 - 64}{3} = \frac{32}{3}$

EXERCISES 4.6

In exercises 5–30, evaluate the indicated integral.

6. $\int \sqrt{1+10x}\,dx$

Let $u = 1+10x$

$du = 10\,dx \Rightarrow dx = \frac{du}{10}$

$= \int \sqrt{u}\,\frac{du}{10} = \frac{1}{10}\int \sqrt{u}\,du = \frac{1}{10}\int u^{\frac{1}{2}}du$

$= \frac{1}{10}\cdot\frac{2}{3}u^{\frac{3}{2}} + C = \frac{1}{15}u^{\frac{3}{2}} + c = \frac{1}{15}(1+10x)^{\frac{3}{2}} + c$

7. $\int \frac{\sin x}{\sqrt{\cos x}}\,dx$

Let $u = \cos x \Rightarrow du$

$du = -\sin x\,dx$

$= \int \frac{1}{\sqrt{u}}(-du) = -\int u^{-\frac{1}{2}}du = -2u^{\frac{1}{2}} + c$

$= -2(\cos x)^{\frac{1}{2}} + c$

10. $\int \sin t(\cos t + 3)^{3/4}dt$

Let $u = \cos t + 3$

$du = -\sin t\,dt$

$= \int (\cos t+3)^{\frac{3}{4}}\underbrace{\sin t\,dt}_{-du} = -\int u^{\frac{3}{4}}\,du$

$= -\frac{4}{7}u^{\frac{7}{4}} + c = -\frac{4}{7}(\cos t+3)^{\frac{7}{4}} + c$

12. $\int e^x \sqrt{e^x + 4}\, dx$

$\overbrace{e^x \cdots dx}^{du}$

$u = e^x + 4$

$du = e^x\, dx$

$= \int \sqrt{u}\ du = \int \sqrt{u}\, du = \int u^{\frac{1}{2}} du$

$= \frac{2}{3} u^{\frac{3}{2}} + c = \frac{2}{3}(e^x + 4)^{\frac{3}{2}} + c$

14. $\int \frac{\cos(1/x)}{x^2}\, dx$

$u = \frac{1}{x} \Rightarrow du = \frac{-1}{x^2}\, dx$

$= \int \cos\left(\frac{1}{x}\right) \frac{dx}{x^2} = \int \cos u\ (-du)$

$= -\int \cos u\ du = \quad = -\sin u + C$

$= -\sin \frac{1}{x} + c$

16. $\int \sec^2 x \sqrt{\tan x}\, dx$

$u = \tan x$

$du = \sec^2 x\, dx$

$= \int \sqrt{\tan x}\ \sec^2 x\, dx = \int \sqrt{u}\ du$

$= \int u^{\frac{1}{2}}\ du = \frac{2}{3} u^{\frac{3}{2}} + C$

$= \frac{2}{3}(\tan x)^{\frac{3}{2}} + c$

17. $\int \frac{1}{\sqrt{u}\,(\sqrt{u}+1)}\,du$

$\cancel{t} = \sqrt{u}+1 = u^{\frac{1}{2}}+1$

$d\cancel{t} = \frac{1}{2}\cancel{u}^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}}\,du$

$2dt = \frac{du}{\sqrt{u}}$

$= \int \frac{1}{(\sqrt{u}+1)}\,\frac{du}{\sqrt{u}}$

$= 2\int \frac{1}{t}\,dt = 2\ln|t| + c = 2\ln|\sqrt{u}+1| + c$

19. $\int \frac{4}{x(\ln x+1)^2}\,dx$

$u = \ln x + 1$

$du = \frac{1}{x}\,dx$

$= 4\int \frac{1}{(\ln x+1)^2}\,\frac{dx}{x} = 4\int \frac{1}{u^2}\,du$

$= 4\int u^{-2}\,du = 4\,\frac{u^{-1}}{-1} + c$

$= -4u^{-1} + c = \frac{-4}{u} + c$

$= \frac{-4}{(\ln x+1)} + c$

In exercises 5–30, evaluate the indicated integral.

22. $\int x^2 \sec^2 x^3 dx$

$u = x^3 \Rightarrow du = 3x^2 dx$

$\frac{du}{3} = x^2 dx$

$= \int \sec^2 x^3 \; x^2 dx = \frac{1}{3}\int \sec^2 u \, du$

$= \frac{1}{3} \tan u + C = \frac{1}{3} \tan x^3 + C$

25. (a) $\int \frac{1+x}{1+x^2} dx = \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx$

$\int \frac{1}{1+x^2} dx$

$= \tan^{-1} x + c$

$\int \frac{x}{1+x^2} dx$

$u = 1+x^2$

$du = 2x dx$

$\frac{du}{2} = x dx$

$= \int \frac{1}{u} \frac{du}{2}$

$= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2} \ln u + c = \frac{1}{2}\ln|1+x^2|$

$\int \frac{1+x}{1+x^2} dx = \tan^{-1} x$

(b) $\int \frac{1+x}{1-x^2} dx = \int \frac{1+x}{(1+x)(1-x)} dx$

$= \int \frac{dx}{1-x}$

$u = 1-x \Rightarrow du = -dx$

$= \int \frac{1}{u}(-du) = -\int \frac{1}{u} du = -\ln|u| + c$

$= -\ln|1-x| + c$

28. $\int \frac{t^2}{\sqrt[3]{t+3}}\,dt$

$u = t+3 \qquad du = dt$

$t = u-3$

$= \int \frac{(u-3)^2}{u^{\frac{1}{3}}}\,du = \int \frac{u^2-6u+9}{u^{\frac{1}{3}}}\,du$

$= \int \frac{u^2}{u^{\frac{1}{3}}}\,du - 6\int \frac{u}{u^{\frac{1}{3}}}\,du + 9\int \frac{1}{u^{\frac{1}{3}}}\,du = \int u^{\frac{5}{3}}\,du - 6\int u^{\frac{2}{3}}\,du - 9\int u^{-\frac{1}{3}}\,du$

$= \frac{3}{8}u^{\frac{8}{3}} - \frac{18}{5}u^{\frac{5}{3}} + \frac{18}{2}u^{\frac{2}{3}} + c = \frac{3}{8}(t+3)^{\frac{8}{3}} - \frac{18}{5}(t+3)^{\frac{5}{3}} + \frac{18}{2}(t+3)^{\frac{2}{3}} + c$

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29. $\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx$

$u = \sqrt{1+\sqrt{x}} \Rightarrow u^2 = 1+\sqrt{x}$

$u^2-1 = \sqrt{x} \Rightarrow (u^2-1)^2 = x$

$dx = 2(u^2-1)(2u)\,du$

$= \int \frac{4u(u^2-1)}{u}\,du = 4\int (u^2-1)\,du$

$= 4\left(\frac{u^3}{3} - u\right) + c = \frac{4}{3}\left(\sqrt{1+\sqrt{x}}\right)^3 - 4\left(\sqrt{1+\sqrt{x}}\right) + c$

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30. $\int \frac{1}{x\sqrt{x^4-1}}\,dx$

$u = x^2 \Rightarrow du = 2x\,dx$

$dx = \frac{du}{2x}$

$= \int \frac{1}{x\sqrt{x^4-1}}\,\frac{dx}{x} = \frac{1}{2}\int \frac{1}{\underbrace{x\sqrt{u^2-1}\;x}_{x^2}}\,du = \frac{1}{2}\int \frac{du}{u\sqrt{u^2-1}}$

$= \frac{1}{2}\sec^{-1}u + c = \frac{1}{2}\sec^{-1}x^2 + C$