

مراجعة الفصل الأول

Question 1: (3+2+1=6 marks)

(a) Solve each of the following inequalities:

(1) $6 \leq 6 - 3x < 18$:

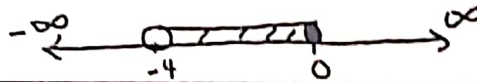
$$6 \leq 6 - 3x < 18$$

$$6 - 6 \leq -3x < 18 - 6$$

$$0 \leq -3x < 12$$

$$\div (-3)$$

$$0 \geq x > -4 \Rightarrow S.T = (-4, 0]$$



(2) $x^2 - 2x - 3 \geq 0$

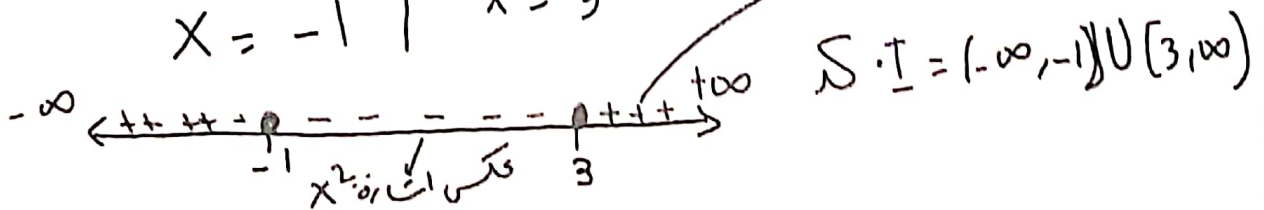
$$(x + 1)(x - 3) \geq 0$$

$$x + 1 = 0$$

$$x = -1$$

$$x - 3 = 0$$

$$x = 3$$

نفس الاستدلال x^2 

(b) Show that the function $f(x) = x^7 - 7$ is one-to-one from $\mathbb{R}$ to $\mathbb{R}$, and find its inverse.

$$f(x_1) = f(x_2)$$

$$x_1^7 - 7 = x_2^7 - 7$$

$$x_1^7 = x_2^7 \Rightarrow x_1 = x_2$$

$f(x)$ is one to one

$$f(x) = x^7 - 7$$

$$y = x^7 - 7$$

$$x = y^7 - 7$$

$$x + 7 = y^7$$

$$y = \sqrt[7]{x+7} \Rightarrow f^{-1}(x) = \sqrt[7]{x+7}$$

(c) Show that the real function $g(x) = x^4$ has no inverse on $\mathbb{R}$.

$$g(x_1) = g(x_2)$$

$$x_1^4 = x_2^4$$

$$x_1 = \pm x_2$$

Not one to one
no inverse

Question 2: (2+2+3=7 marks)

(a) Find the distance between two points (x_1, y_1) and (x_2, y_2) , and an equation of the line through them.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(4-2)^2 + (3-1)^2} = \sqrt{4+4}$$

$$d = \sqrt{8}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{3-1} = \frac{2}{2} = 1$$

$$y = m(x - x_1) + y_1$$

$$y = 1(x - 1) + 2$$

$$y = x - 1 + 2$$

$$y = x + 1$$

(b) Find an equation of the line perpendicular $\perp$ to the line $y = -2x + 4$ and intersecting this line at the point $(1, 2)$.

الخطي

$$m_1 = -2 \Rightarrow m_2 = \frac{1}{2}$$

$$y = m(x - x_1) + y_1$$

$$y = \frac{1}{2}(x - 1) + 2 \Rightarrow y = \frac{1}{2}x - \frac{1}{2} + 2$$

$$y = \frac{1}{2}x + \frac{3}{2}$$

(c) Find the domain of $f(x) = \frac{x^3 + 5x - 6}{\sqrt{x^2 - 16}}$;

$$x^2 - 16 > 0$$

$$(x - 4)(x + 4) > 0$$

$$x - 4 = 0 \quad | \quad x + 4 = 0$$

$$x = 4 \quad | \quad x = -4$$

$$\leftarrow \begin{array}{c} + + + \\ 0 \end{array} \quad \begin{array}{c} - - - \\ 0 \end{array} \quad \begin{array}{c} + + + \\ 0 \end{array} \rightarrow$$

$$S.I = (-\infty, -4) \cup (4, \infty)$$

find the zeros of $h(x) = \sqrt{x^2 - 5x - 12}$.

$$x^2 - 5x - 12 = 0$$

$a = 1 \quad b = -5 \quad c = -12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Zeros} \Rightarrow x = \frac{-5 \pm \sqrt{73}}{2}$$

$$x = \frac{5 \pm \sqrt{25 - 4(1)(-12)}}{2}$$

$$x = \frac{-5 \pm \sqrt{25 + 48}}{2}$$

$$x = \frac{-5 \pm \sqrt{73}}{2}$$

$$x = \frac{-5 - \sqrt{73}}{2}$$

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Question 3: (2.5+1.5+2=6 marks)

(a) Find all solutions of the equation: $\sin^2 x - 3 \sin x + 2 = 0$.

$$(\sin x - 1)(\sin x - 2) = 0$$

$$\sin x - 1 = 0 \quad \left| \quad \sin x - 2 = 0 \right.$$

$$\sin x = 1$$

$$x = \frac{\pi}{2}$$

$$\sin x = 2 \quad (\text{No})$$

$$-1 \leq \sin x \leq 1$$

(b) Derive the identity $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$.

$$\begin{aligned} \cos 2\theta &= \cos(\theta + \theta) = \cos \theta \cos \theta - \sin \theta \sin \theta \\ &= \cos^2 \theta - \sin^2 \theta \end{aligned}$$

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

(c) Evaluate $\sin^{-1}(-0.5)$; and simplify $\cos(\tan^{-1} x)$

$$\sin^{-1}\left(-\frac{1}{2}\right)$$

$$\sin \theta = -\frac{1}{2}$$

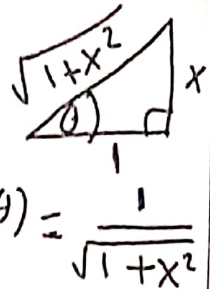
$$\theta = -\frac{\pi}{6} \text{ or } -\frac{5\pi}{6}$$

x	$\sin x$	$\cos x$
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
π	0	-1
$\frac{3\pi}{2}$	-1	0
2π	0	1

$$\cos(\tan^{-1} x)$$

$$\theta = \tan^{-1} x$$

$$\frac{\text{y. قاطع}}{\text{x. قاطع}} = \frac{x}{1} = \tan \theta$$



$$\cos(\tan^{-1} x) = \cos(\theta) = \frac{1}{\sqrt{1+x^2}}$$

Question 4: (1.5+1.5+3=6 marks)

(a) Convert to exponential form $(3^x \cdot 3^{3+x})^3$; and rewrite $(2/5)^x$ as exponentials with base e

$$(3^x \cdot 3^{3+x})^3 =$$

$$\left(\frac{2^{2x+3}}{3}\right)^3 =$$

$$= 3^{6x+9}$$

$$\left(\frac{2}{5}\right)^x = e^{\ln\left(\frac{2}{5}\right)^x}$$

$$= e^{x \ln\left(\frac{2}{5}\right)}$$

(b) Determine

$$\log(10000) = \frac{\log 10^4}{10} = 4$$

$$\log(0.0001) = \frac{\log 10^{-4}}{10} = -4$$

$$\frac{1}{10000} = \frac{1}{10^4} = 10^{-4}$$

$$\ln(e^5) = 5$$

$$\underbrace{\ln e}_{\ln e} = 1$$

$$\underbrace{e}_{e} = 1$$

(c) Solve each of the following equations

$$3e^x - 2 = 10$$

$$3e^x = 10 + 2$$

$$3e^x = 12 \quad \div (3)$$

$$e^x = 4 \quad \leftarrow \ln$$

$$\ln e^x = \ln 4 \Rightarrow x = \ln 4$$

$$\ln(x-5) = 2$$

$$\ln(x-5) = e^2$$

$$x-5 = e^2$$

$$x = e^2 + 5$$

Bonus [2]: Write $\log_2 27^x - \log_2 3^x$ as a single logarithm.

$$\begin{aligned}
 &= \log_2 \frac{27^x}{3^x} \quad \text{الباقي} \\
 &= \log_2 \frac{(3^3)^x}{3^x} \\
 &= \log_2 \frac{3^{3x}}{3^x} = \log_2 3^{3x-x} \\
 &= \log_2 3^{2x}
 \end{aligned}$$

expand $\ln \left(\frac{x^4 y^5}{z^3} \right)$.

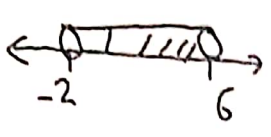
$$\begin{aligned}
 &= \ln(x^4 y^5) - \ln z^3 \quad \text{الباقي} \\
 &= \ln x^4 + \ln y^5 - \ln z^3 \\
 &= 4 \ln x + 5 \ln y - 3 \ln z
 \end{aligned}$$

$$\begin{aligned}
 &\ln a^b \\
 &= b \ln a
 \end{aligned}$$

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Question 1. [5*2=10 marks]

1) Solve the inequality $|2x - 4| < 8$.

$$\begin{aligned}
 & -8 < 2x - 4 < 8 \\
 & -8 + 4 < 2x < 8 + 4 \\
 & -4 < 2x < 12 \quad (\div 2) \\
 & -2 < x < 6
 \end{aligned}$$


2) Find the domain of the function $f(x) = \frac{4}{x^2 - 9}$.

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$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

$$D_f = \mathbb{R} - \{ \pm 3 \} =$$

$$= (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$$

3) Find the equation of a line through the points $(7, -2)$ and $(2, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-2)}{2 - 7} = \frac{4}{-5}$$

$$\begin{matrix} x_1 & y_1 \\ (7, & -2) \end{matrix}$$

$$y = m(x - x_1) + y_1$$

$$y = \frac{-4}{5}(x - 7) - 2$$

4) Find the inverse of $f(x) = \sqrt[5]{x^3 + 1}$.

$$\begin{aligned}
 & \checkmark y = \sqrt[5]{x^3 + 1} \\
 & \text{sw. } x = \sqrt[5]{y^3 + 1} \\
 & \left| \begin{aligned}
 & x^5 = y^3 + 1 \\
 & x^5 - 1 = y^3 \Rightarrow y = \sqrt[3]{x^5 - 1} \\
 & f^{-1}(x) =
 \end{aligned}
 \right.
 \end{aligned}$$

5) Show that $f(g(x)) = x$ and $g(f(x)) = x$ for all x so that $f^{-1} = g$ for

$$f(x) = (4x^3) \text{ and } g(x) = \left(\frac{1}{4}x\right)^{\frac{1}{3}}$$

$$\begin{aligned}
 f(g(x)) &= 4\left(\left(\frac{1}{4}x\right)^{\frac{1}{3}}\right)^3 \\
 &= 4 \cdot \frac{1}{4}x = x \\
 g(f(x)) &= \left(\frac{1}{4}(4x^3)\right)^{\frac{1}{3}} \\
 &= (x^3)^{\frac{1}{3}} = x
 \end{aligned}$$

$$f^{-1} = g$$

Question 2. [2*2+2*1=6 marks]

1) Find all the solutions of the given equations:

(a) $2 \sin x + 1 = 0$,

$2 \sin x + 1 = 0 \Rightarrow 2 \sin x = -1$

$\sin x = -\frac{1}{2} \Rightarrow x = -\frac{\pi}{6}, -\frac{5\pi}{6}$

$x = -\frac{\pi}{6} + 2k\pi, x = -\frac{5\pi}{6} + 2k\pi$

x	$\sin x$	$\cos x$
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
π	0	-1
$\frac{3\pi}{2}$	-1	0
2π	0	1

(b) $x e^{-2x} + 2 e^{-2x} = 0$.

$e^{-2x} (x+2) = 0$

$e^{-2x} \neq 0$ $\Rightarrow x+2 = 0$
 $0 = \dots$ $\Rightarrow x = -2$

2) Evaluate

(a) the inverse cosine function $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

$$\cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$$

(b) the expression $3 \ln 2 - \ln \frac{1}{2}$.

$$③ \ln 2 - \ln \frac{1}{2} =$$

$$\ln 2^3 - \ln \frac{1}{2} = \ln 2^3 - \ln 2^{-1}$$

$$\begin{aligned} \ln 2^3 + \ln 2 &= \ln 2^3 \cdot 2^1 \\ &= \ln 2^4 \end{aligned}$$

x	$\sin x$	$\cos x$
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
π	0	-1
$\frac{3\pi}{2}$	-1	0
2π	0	1

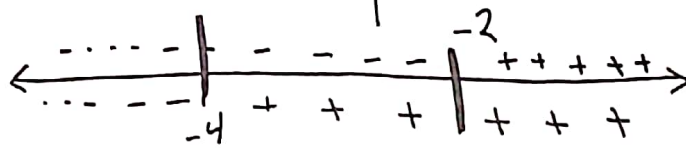
مراجعة الفصل الأول

Question 1. [4*2=8 marks]

1) Solve the inequality $\frac{x^2 + 2}{x + 4} \geq 0$.

$$x + 2 = 0 \quad | \quad x = -2$$

$$x + 4 = 0 \quad | \quad x = -4$$



$$S.I = (-\infty, -4) \cup [-2, \infty)$$

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2) Find the domain of the function $f(x) = \frac{1}{\sqrt{4 - x^2}}$.

المقام > 0

$$4 - x^2 > 0$$

$$4 > x^2 \Rightarrow 2 > |x|$$

$$-2 < x < 2$$

$$D_f = (-2, 2)$$

3) Find an equation of the line perpendicular to $y = -3x + 2$ and intersecting the line at the point $(1, -1)$.

محدودى

$$m_1 = -\frac{3}{1}$$

$$m = \frac{1}{3}$$

$$x_1, y_1$$

$$(1, -1)$$

$$y = m(x - x_1) + y_1$$

$$y = \frac{1}{3}(x - 1) - 1 \Rightarrow y = \frac{1}{3}x - \frac{1}{3} - 1$$

$$-\frac{1}{3} - 1 = -\frac{1}{3} - \frac{3}{3} = -\frac{4}{3}$$

$$y = \frac{1}{3}x - \frac{4}{3}$$

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4) Show that the function $f(x) = 5x^5 + 2$ is one-to-one and find its inverse.

$$f(x_1) = f(x_2)$$

$$5x_1^5 + 2 = 5x_2^5 + 2$$

$$5x_1^5 = 5x_2^5$$

$$x_1 = x_2$$

f is one to one

$$f(x) = 5x^5 + 2$$

$$y = 5x^5 + 2$$

$$x = \sqrt[5]{\frac{y-2}{5}}$$

$$\frac{x-2}{5} = \frac{y}{5}$$

$$\frac{x-2}{5} = y^5$$

$$y = \sqrt[5]{\frac{x-2}{5}} = f^{-1}(x)$$